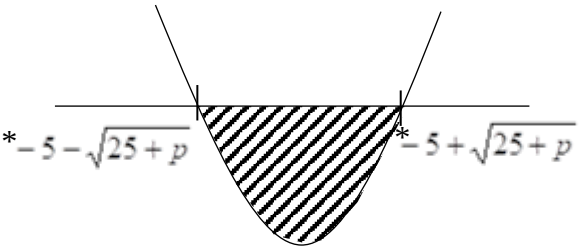
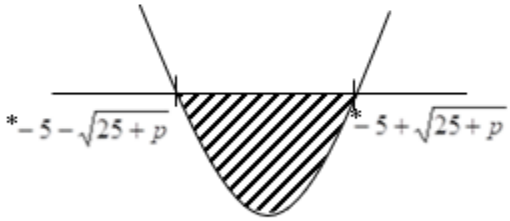
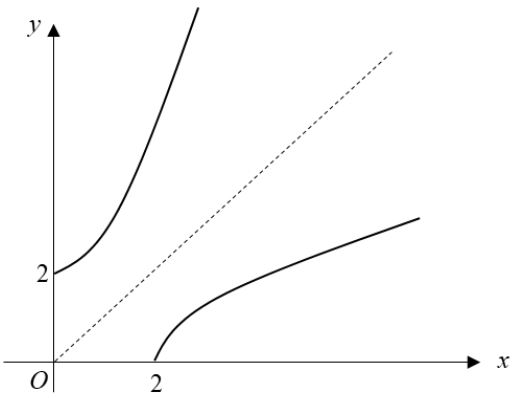
Additional Mathematics Paper 1
SPMRSM 2021
ANSWER SCHEME

No	Solution	Scheme	Sub marks	Marks
1	$8\sqrt{a}$ **or** $3\sqrt{a}$ $11\sqrt{a}-\frac{3}{\sqrt{a}}$ **or equivalent** $^{*}\frac{11a-3}{\sqrt{a}}\times\left(\frac{\sqrt{a}}{\sqrt{a}}\right)$ **or equivalent** $\frac{(11a-3)\sqrt{a}}{a}$	P1 Seen **or** Implied rationalizing K1 K1 simplifying N1		4
2 (a)	$5^{x}\left(5^{1}\right)$ or $5^{x}\left(5^{3}\right)$ $42x+5=5+5^{3}+1$ 3	P1 Seen **or** Implied K1 Comparing and solve N1	3	
(b)	$\log_{m}\left(\sqrt{mn}\times\frac{m}{\sqrt{n}}\right)$ $\frac{3}{2}\log_{m} m$ $\frac{3}{2}$	Using $\log_a mn = \log_a m + \log_a n$ **or** $\log_a \frac{m}{n} = \log_a m - \log_a n$ K1 K1 Using $\log_a m^n = n \log_a m$ N1	3	6

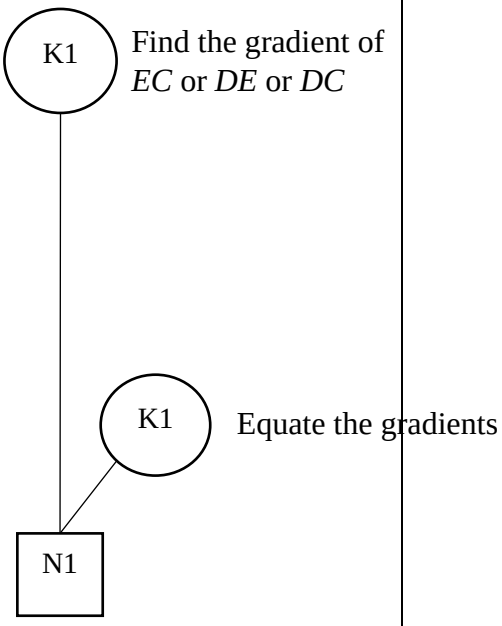
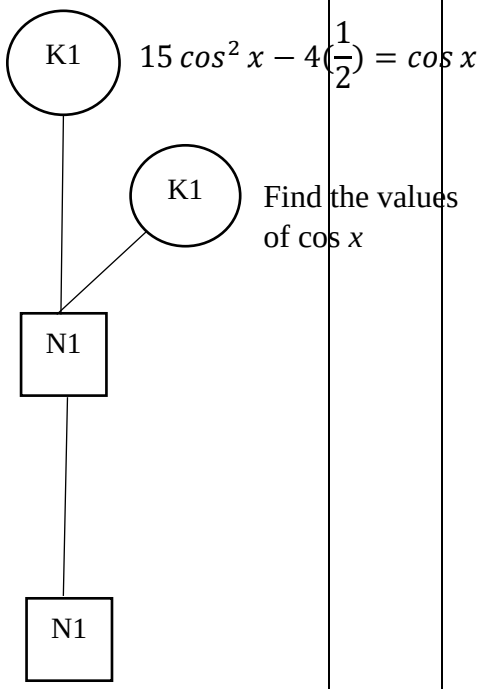
3 (a)	$x^2y + 5(5x^2) = 4500$ **or** $\frac{4}{2}\left[2(x^2y) + 3(5x^2)\right] = 11000$ **or** $x^2y + x^2(y+5) + x^2(y+10) + x^2(y+15) = 11000$ $\frac{x^2(2y+15)}{x^2(y+25)} = \frac{5500}{4500}$ $y = 20$ $x = 10$	K1 *Use* $T_6 = a + 5d$ *OR* $S_4 = \frac{4}{2}\left[2a + (n-1)d\right]$ or $a + T_2 + T_3 + T_4$ K1 Solve simultaneous eqn using correct method N1 Both correct $y = 20$, $x = 10$	3	
(b)	$100(^*20) + 9(500)$ **or** $\frac{6500}{100}$ or $^*20 + 9(5)$ **OR** $^*20 + (n-1)(5) \leq 60$ (Accept equal sign) No	K1 Find T_{10} or <u>height</u> for T_{10} N1 65 or $n \leq 9$ or n=9 must be seen	2	5

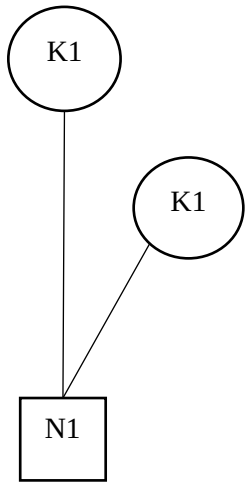
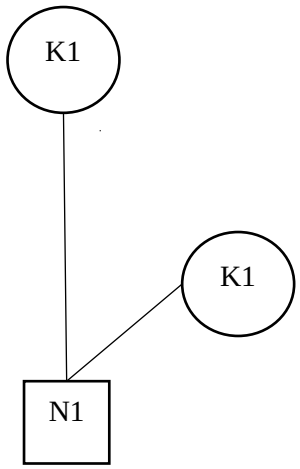
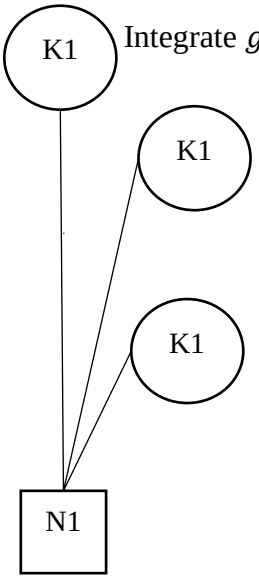
4 (a)	$x(x+10) \le p$ $x^2+10x-p \le 0$ $\left(x+\frac{10}{2}\right)^2-\left(\frac{10}{2}\right)^2-p \le 0$ **or** $x=\frac{-10\pm\sqrt{(10)^2-4(1)(-p)}}{2(1)}$ $x=-5+\sqrt{25+p}\,,\qquad x=-5-\sqrt{25+p}$  $-5-\sqrt{25+p} \le x \le -5+\sqrt{25+p}$	K1 Form a quadratic inequality K1 Use valid method to solve for quadratic inequality K1 Use valid method to determine correct region Accept $x=\frac{-10\pm\sqrt{(10)^2-4(1)(-p)}}{2(1)}$ N1 $-5-\sqrt{25+p} \le x \le -5+\sqrt{25+p}$	4	
(b)	Two real and different roots	P1	1	5

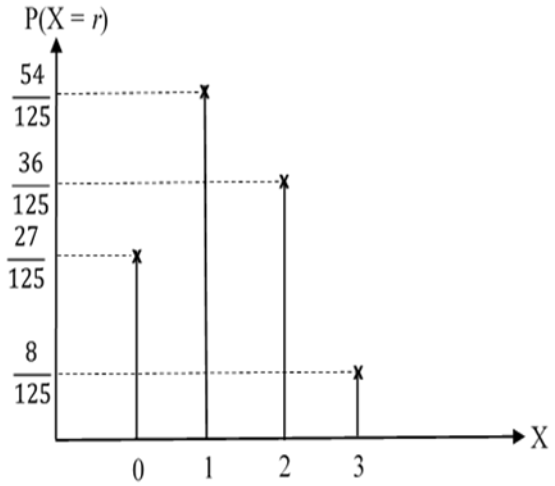
4 (a)	$x(x+10) \le p$ $x^2+10x-p \le 0$ $\left(x+\frac{10}{2}\right)^2-\left(\frac{10}{2}\right)^2-p \le 0$ **or** $x=\frac{-10\pm\sqrt{(10)^2-4(1)(-p)}}{2(1)}$ $x=-5+\sqrt{25+p}, \quad x=-5-\sqrt{25+p}$  $-5-\sqrt{25+p} \le x \le -5+\sqrt{25+p}$ $0 < AQ \le -5+\sqrt{25+p}$	K1 K1 K1 N1	Form a quadratic inequality Use valid method to solve for quadratic inequality Use valid method to determine correct region Accept $x=\frac{-10\pm\sqrt{(10)^2-4(1)(-p)}}{2(1)}$	4

5 (a)	i) {1, 2, 4, 5, 7} ii) {2, 3, 5, 6, 8}	N1 N1	2	
(b)	**CASE 1** $f^{-1}(x) = \frac{x+7}{3}$ $g^{-1}(x) = \frac{4x-8}{x}$ $f^{-1}g^{-1}(x) = \left[\left(\frac{4x-8}{x}\right)+7\right]^{*}{}^{*}$ $\frac{11x-8}{3x}, x \neq 0$ or $\frac{11}{3} - \frac{8}{3x}, x \neq 0$	K1 finding $f^{-1}(x)$ **or** $g^{-1}(x)$ K1 N1	3	
	CASE 2 $gf(x) = \frac{8}{4-(3x-7)}$ $= \frac{8}{11-3x}, x \neq \frac{11}{3}$ $(gf)^{-1} = \frac{11x-8}{3x}, x \neq 0$	K1 N1 N1		
(c)	 (0, 2)	P1 P1	2	7

6 (a)	(i) $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$	N1	1	
	(ii) $\sqrt{8^2+6^2}$	K1		
	$\frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$	N1 $\frac{1}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ Note: Accept without working	2	
(b)	$(4\ ,3)$	P1 Seen **or** implied		
	$\frac{r(h-k)+4k}{h}=-2$ **or** $\frac{-4(h-k)+3k}{h}=0$	K1		
	$4h=7k$	K1 Solving simultaneous equations		
	$\frac{r(3)+4k}{h}=-2$	N1		
	$r=-10$		4	7

	ALTERNATIVE METHOD $\frac{3-0}{4-(-2)}$ *or* $\frac{0-(-4)}{-2-r}$ *or* $\frac{3-(-4)}{4-r}$ $\frac{3-0}{4-(-2)} = \frac{0-(-4)}{-2-r}$ or equivalent $r = -10$			
7	$15\,cos^2 x - 4(\frac{1}{2}) = cos\, x$ $(5\cos x - 2)(3\cos x + 1) = 0$ $\cos x = \frac{2}{5}$, $\cos x = \frac{-1}{3}$ $66.42°, 70.53°$ // $66°25', 70°32'$ or $66.42°, 109.47°$ //$66°25', 109°28'$ $66.42°, 109.47°, 250.53°, 293.58°$ // $66°25', 109°28', 250°32', 293°35'$			4

8	$\int \frac{(2x+1)^2(4x-19)}{(x-3)^2}dx = \frac{(2x+1)^3}{x-3}$ $\frac{2}{3}\left[\frac{(2x+1)^3}{x-3}\right]$ $*\frac{2}{3}\left[\frac{(2(5)+1)^3}{5-3} - \frac{(2(4)+1)^3}{4-3}\right]$ $-\frac{127}{3}$ // $-42\frac{1}{3}$	P1 Seen **or** implied  Use limit 4, 5		4
9 (a)	$\frac{\left(2-\frac{1}{4}x\right)^{-5+1}}{(-5+1)\left(-\frac{1}{4}\right)}$ $\frac{\left(2-\frac{1}{4}(4)\right)^{-5+1}}{(-5+1)\left(-\frac{1}{4}\right)} - \frac{\left(2-\frac{1}{4}(m)\right)^{-5+1}}{(-5+1)\left(-\frac{1}{4}\right)} = \frac{65}{81}$ 2	 Use limit m, 4 and equate to $\frac{65}{81}$, index must increase by 1	3	
(b)	$g'(x) = -4x + c$ $*[-4(1) + c_1] = -8$ $c_1 = -4$ $*[-4x - 4] = 0$ $x = -1$ $g(x) = -2x^2 - 4x + 8$	 	4	7

10 (a)	(i) 5040 (ii) $4! \times 4!$ **or** ${}^4P_4 \times {}^4P_4$ **or** ${}^4P_4 \times {}^4P_2 \times {}^2P_2$ 576	N1 K1 N1	3	
(b)	${}^3C_1 \times {}^2C_2$ **or** ${}^3C_2 \times {}^2C_1$ **or** ${}^3C_3 \times {}^2C_0$ $({}^3C_1 \times {}^2C_2) + ({}^3C_2 \times {}^2C_1) + ({}^3C_3 \times {}^2C_0)$ 10	K1 K1 N1	3	6
11 (a)	${}^nC_0(0.4)^0(0.6)^n = \frac{27}{125}$ **or** ${}^nC_1(0.4)^1(0.6)^{n-1} = \frac{54}{125}$ **or** equivalent $n = 3$	K1 N1	Use ${}^nC_r\,(0.4)^r\,(0.6)^{n-r}$ and solve	2	
(b)		P1 P1	Correct shape of binomial graph with their n Ignore the values of $P(X = r)$ All correct	2	4

12	$\log_{10} y = (\log_{10} h)x - \log_{10}\sqrt{k}$ $\log_{10} h = \dfrac{\frac{17}{2}-\frac{9}{2}}{4-2}$ $h = 100$ $-\dfrac{1}{2}\log_{10} k = \dfrac{9}{2} - \left(\dfrac{\frac{17}{2}-\frac{9}{2}}{4-2}\right)(2)$ $k = \dfrac{1}{10}$ // 0.1	P1 Implied K1 N1 K1 N1		5
13	$2p + 2q = 100$ $p = 50 - q$ $2\pi r = p$ $r = \dfrac{p}{2\pi}$ **CASE 1** $V = \pi r^2 h$ $= \pi\left(\dfrac{p}{2\pi}\right)^2 q$ $= \pi\left(\dfrac{50-q}{2\pi}\right)^2 q$ $= \dfrac{625q}{\pi} - \dfrac{25q^2}{\pi} + \dfrac{q^3}{4\pi}$ $\dfrac{dV}{dq} = \dfrac{1}{\pi}\left(625 - 50q + \dfrac{3q^2}{4}\right)$ $\dfrac{1}{\pi}\left(625 - 50q + \dfrac{3q^2}{4}\right) = 0$ $(q-50)(3q-50) = 0$ $q = 50, \quad q = \dfrac{50}{3}$ $p = \dfrac{100}{3}$	P1 2p + 2q = 100 or 2πr = p K1 Find V in term of q K1 Find $\dfrac{dV}{dq}$ K1 Use $\dfrac{dV}{dq} = 0$ to find q N1		

13	$2p+2q=100$ $p=50-q$ $2\pi r=p$ $r=\frac{p}{2\pi}$ **CASE 1** $V=\pi r^2 h$ $=\pi\left(\frac{p}{2\pi}\right)^2 q$ $=\pi\left(\frac{50-q}{2\pi}\right)^2 q$ $=\frac{625q}{\pi}-\frac{25q^2}{\pi}+\frac{q^3}{4\pi}$ $\frac{dV}{dq}=\frac{1}{\pi}\left(625\ -50q+\frac{3q^2}{4}\right)$ $\frac{1}{\pi}\left(625\ -50q+\frac{3q^2}{4}\right)=0$ $(q-50)(3q-50)=0$ $q=50,\ q=\frac{50}{3}$ $p=\frac{100}{3}$	P1	$2p + 2q = 100$ or $2\pi r = p$	
		K1	Find V in term of q	
		K1	Find $\frac{dV}{dq}$	
		K1	Use $\frac{dV}{dq}=0$ to find q	
		N1		

CASE 2

$V = \pi r^2 h$

$= \pi\left(\frac{p}{2\pi}\right)^2 q$

$= \pi\left(\frac{p}{2\pi}\right)^2 (50 - p)$

$= \frac{50}{4\pi} p^2 - \frac{1}{4\pi} p^3$

$\frac{dV}{dp} = \frac{50}{4\pi}(2p) - \frac{1}{4\pi}(3p^2)$

$\frac{50}{4\pi}(2p) - \frac{1}{4\pi}(3p^2) = 0$

$p = \frac{100}{3}$

K1 Find V in term of p

K1 Find $\frac{dV}{dp}$

K1 Use $\frac{dV}{dp} = 0$ to find p

N1

$r = \frac{p}{2\pi}$

$= \frac{^*\frac{100}{3}}{2\pi}$

$= 5.305$

$width = 2(5.305)$

$= 10.61$

Yes

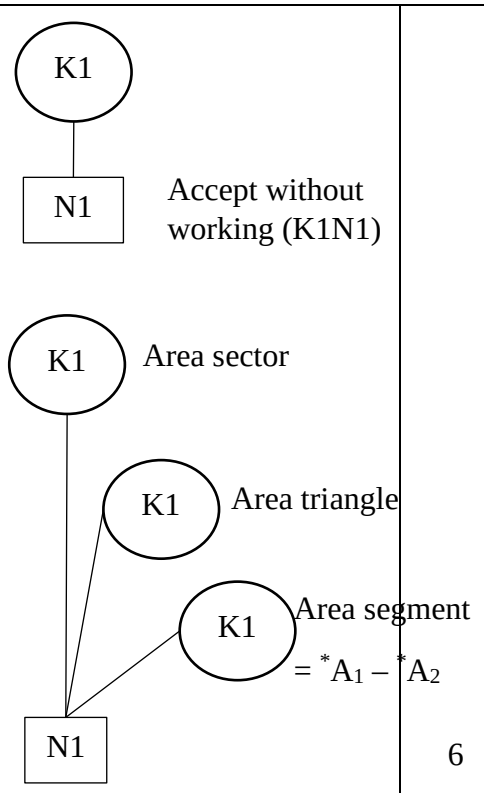
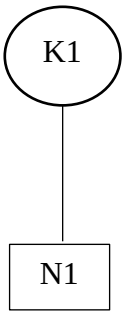
K1 Find the value of r OR circumference of circle

K1 10.61 OR 5.5 OR 34.56 seen

N1

8

14 (a)	(i) $S_n = \frac{n}{2}[2(1)+(n-1)2]$ OR 1, 4, 9, … (at least 3 terms listed) $S_1 = 1,\ S_2 = 4,\ S_3 = 9$ $S_n = n^2$	K1 N1 $S_n = n^2$	2	
	(ii) $S_{25} - S_{19} = 25^2 - 19^2$ 264	K1 Use $S_n = {}^{*}n^2$ and find $S_{25} - S_{19}$ N1	2	
(b)	(i) 785,778.72,772.49, ... $a = 785$ $r = 0.992$ $T_n = 785(0.992)^{n-1}$	N1	1	
	(ii) $785(0.992)^{n-1} \le \frac{785}{2}$ $(n-1)\,log_{10}\,0.992 \le log_{10}\,0.5$ $n - 1 \ge \frac{log_{10}\,0.5}{log_{10}\,0.992}$ $n \ge 87.296$ $n = 88$	K1 Note: Accept "<" and "=" K1 Use logarithm to solve N1	3	8

15 (a)	$\pi = 3\theta$ $\theta = \frac{\pi}{3}$ $A_1 = \frac{1}{2}(3)^2\left(\frac{\pi}{3}\right)$ $A_2 = \frac{1}{2}(3)^2 \sin\left(\frac{\pi}{3}\right)$ Area segment = $(A_1 - A_2)$ Total area segment = 14 $(A_1 - A_2)$ $11.40 \leq Area \leq 11.42$			
(b)	$\frac{\text{RM}1.20\times^{*}11.41}{45}\times 100\%$ $30.40\% \leq percentage \leq 30.46\%$		6 2	8